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Tensor properties of added-mass and damping coefficients

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Abstract. It has been shown that in the context of a linear theory for a floating body with six degrees of freedom each of the 6×6 added-mass and damping matrices contains three distinct Cartesian second-order tensors in regard to translational, rotational and interaction between translational and rotational oscillations. As a result of this, a new technique based on the transformation law of second-order tensors is introduced for motion analysis of offshore platforms which can be used as an alternative to the common methods in offshore engineering.

Key words: floating bodies, transformation method, truss spar

1. Introduction

In marine hydrodynamics, like other branches of continuum mechanics, it is customary to use index notation and summation convention when writing equations in a compact form. Since a marine vehicle is usually assumed to be a rigid body, and a rigid body in a three-dimensional space generally has six degrees of freedom, the range of indices in marine hydrodynamics is assumed to be 1 to 6, rather than the usual range of 1 to 3. This range convention helps to write equations in a very compact form, but sometimes the resulted compactness hides some valuable information. One of the important aspects, which is hidden and ignored due to the traditional range convention of marine hydrodynamics, is the tensor character of added-mass and damping coefficients of immersed and floating bodies. If $m_{\alpha\beta}$ denotes the added-mass coefficients of an immersed body, where α and β as usual range 1 to 6, it is shown in Section 2 that $m_{\alpha\beta}$ contain three distinct Cartesian second-order tensors in three-dimensional space.

In the study of tensor properties of suspension particles, Happel and Brenner [1, Chapter 5] obtained similar tensors. Their study is limited to the case of a rigid particle immersed in an unbounded fluid, whose results can be used for an immersed marine structure. Here the theory is extended to the case of a floating body. As a result, the powerful tools of tensor analysis, which have been used in other branches of mechanics for many years, can now be applied in marine hydrodynamics. An application of this method in the response analysis of a truss spar platform is shown in Section 4.

Throughout this article Greek indices range from 1 to 6, Latin indices range from 1 to 3 and the summation convention is implied for repeated indices. Also, the word "tensor" is used to refer to tensors, pseudo-tensors and some quantities that obey the transformation law of a tensor, and, unless it is explicitly specified, by a "tensor" is meant a tensor in that broad inexact sense. The mathematical background of the stated material can be found in [2–5].

2. Second-order tensors of radiation problem

2.1. Motion in unbounded fluid

We shall start with the radiation problem of an immersed body. It is well-known that the kinetic energy T of the fluid domain can be written as [6]:

$$T = \frac{1}{2} m_{\alpha\beta} U_\alpha U_\beta, \tag{1}$$

where U_i is the translational velocity of the rigid body at an origin fixed to the body, U_{i+3} is the angular velocity of the body and $m_{\alpha\beta}$ are added-mass coefficients of the body. In Equation (1), α and β are dummy indices, it follows that $m_{\alpha\beta}$ are symmetric coefficients. Replacing Greek indices with Latin indices and expanding the right-hand side of this equation, it follows that

$$T = \frac{1}{2}(m_{ij} U_i U_j + m_{i+3,\,j} U_{i+3} U_j + m_{i,\,j+3} U_i U_{j+3} + m_{i+3,\,j+3} U_{i+3} U_{j+3}). \tag{2}$$

If one defines $m_{i+3,\,j+3} = I_{ij}$ and $m_{i,\,j+3} = J_{ij}$, then because $m_{\alpha\beta} = m_{\beta\alpha}$, it follows from the second definition that $m_{i+3,\,j} = J_{ji}$. Therefore, denoting U_{i+3} by Ω_i, Equation (2) can be written as

$$T = \frac{1}{2} m_{ij} U_i U_j + \frac{1}{2} J_{ji} \Omega_i U_j + \frac{1}{2} J_{ij} U_i \Omega_j + \frac{1}{2} I_{ij} \Omega_i \Omega_j\,. \tag{3}$$

Now, since the kinetic energy T on the left-hand side of Equation (3) is a zeroth-order tensor (a scalar) and U_i and Ω_i in each term on the right-hand side of Equation (3) are components of two first-order tensors (two vectors), it follows from the quotient rule that m_{ij}, J_{ij} and I_{ij} must be components of three distinct Cartesian second-order tensors. One can call m_{ij}, J_{ij} and I_{ij} the components of the added-mass, added-product of inertia and added-moment of inertia tensors, respectively. Alternatively, we prefer to call them the components of the zeroth-moment, first-moment and second-moment added-mass tensors, respectively. For each tensor, we shall use both names interchangeably.

2.2. Effect of a free surface

Now we shall consider the linear radiation problem of a floating body. Following Newman [7, pp. 285–300] the generalized radiation force F_α acting on a rigid floating body can be written as

$$F_\alpha = -A_{\alpha\beta} \dot{U}_\beta - B_{\alpha\beta} U_\beta, \tag{4}$$

where $A_{\alpha\beta}$ and $B_{\alpha\beta}$ are added-mass and damping coefficients, respectively, and a dot denotes differentiation with respect to time. Replacing Greek indices with Latin indices and using Ω_i in place of U_{i+3}, we may expand Equation (4) into the following two equations

$$\begin{aligned} F_i &= -A_{ij} \dot{U}_j - A_{i,\,j+3} \dot{\Omega}_j - B_{ij} U_j - B_{i,\,j+3} \Omega_j, \\ M_i &= -A_{i+3,\,j} \dot{U}_j - A_{i+3,\,j+3} \dot{\Omega}_j - B_{i+3,\,j} U_j - B_{i+3,\,j+3} \Omega_j. \end{aligned} \tag{5}$$

Now defining $A_{ij} = \widetilde{m}_{ij}$, $A_{i,\,j+3} = \widetilde{S}_{ij}$, $A_{i+3,\,j+3} = \widetilde{I}_{ij}$, $B_{i,\,j+3} = D_{ij}$, $B_{i+3,\,j+3} = E_{ij}$, and taking into account the symmetry of $A_{\alpha\beta}$ and $B_{\alpha\beta}$ [7, p. 296], we have that Equation (5) takes the following form

$$\begin{aligned} F_i &= -\widetilde{m}_{ij} \dot{U}_j - \widetilde{S}_{ij} \dot{\Omega}_j - B_{ij} U_j - D_{ij} \Omega_j, \\ M_i &= -\widetilde{S}_{ji} \dot{U}_j - \widetilde{I}_{ij} \dot{\Omega}_j - D_{ji} U_j - E_{ij} \Omega_j. \end{aligned} \tag{6}$$

Because on the left-hand side of Equation (6) F_i and M_i are components of the force and moment vectors (two first-order tensors) and on the right-hand side of this equation U_i, $\dot{U}_i$, Ω_i and $\dot{\Omega}_i$ are components of the velocity, acceleration, angular velocity and angular acceleration vectors (four first-order tensors), it follows from the quotient rule that $\widetilde{m}_{ij}$, $\widetilde{S}_{ij}$, $\widetilde{I}_{ij}$, B_{ij}, D_{ij}, and E_{ij}, must be components of six distinct Cartesian second-order tensors. In Equation (6) $\widetilde{m}_{ij}$, $\widetilde{S}_{ij}$ and $\widetilde{I}_{ij}$ are components of the added-mass, the added-product of inertia and the added-moment of inertia tensors of a floating body, respectively. In analogy with m_{ij}, J_{ij} and I_{ij}, we shall call B_{ij}, D_{ij} and E_{ij}, respectively, the components of the zeroth-moment, first-moment and second-moment-damping tensors. One may refer to the nine second-order tensors m_{ij}, J_{ij}, I_{ij}, $\widetilde{m}_{ij}$, $\widetilde{S}_{ij}$, $\widetilde{I}_{ij}$, B_{ij}, D_{ij} and E_{ij} as radiation tensors. We shall refer to three tensors m_{ij}, $\widetilde{m}_{ij}$ and B_{ij} as zeroth-moment-radiation tensors; to three tensors J_{ij}, $\widetilde{S}_{ij}$ and D_{ij} as first-moment-radiation tensors and to I_{ij}, $\widetilde{I}_{ij}$ and E_{ij} as second-moment-radiation tensors. Zeroth- and second-moment-radiation tensors are symmetric tensors. This follows directly from the symmetry of $m_{\alpha\beta}$, $A_{\alpha\beta}$ and $B_{\alpha\beta}$.

By using a similar approach as used here, we can show that for the linear radiation problem of a floating body, in addition to the added-mass and damping matrices, the 6×6 hydrostatic restoring matrix also contains three distinct Cartesian second-order tensors in regard to translational, rotational and interaction between translational and rotational degrees of freedom.

3. Tensor properties of radiation coefficients

Having shown that radiation coefficients $m_{\alpha\beta}$, $A_{\alpha\beta}$ and $B_{\alpha\beta}$ contain nine distinct second-order tensors, we have made available all powerful tools of tensor analysis for the radiation problem of an immersed or a floating body. Some of these tools are related to the problem of obtaining the components of a tensor in one coordinate system when these components are known in another coordinate system. We shall express the rotation, reflection and translation laws for radiation tensors in this section.

3.1. The transformation law of radiation tensors

Consider two right-handed rectangular Cartesian coordinate systems x_i and x'_i with the same origin, where the primed coordinate system x'_i is obtained by rotating the unprimed coordinate system x_i about the common origin. It is known that, if R_{ij} are components of a second-order tensor in the x_i coordinate system, they transform to components R'_{ij} in the x'_i coordinate system by the following transformation law

$$R'_{ij} = a_{ik}\, a_{jl}\, R_{kl}, \tag{7}$$

in which a_{ij} is the direction cosine or the transformation symbol. For $\mathbf{e}'_i$ and $\mathbf{e}_i$, respectively, as unit basis vectors of primed- and unprimed-coordinate systems, a_{ij} is defined as

$$a_{ij} = \mathbf{e}'_i \cdot \mathbf{e}_j = \cos(\widehat{x'_i,\, x_j}). \tag{8}$$

Since it is shown that each of the nine radiation tensors is a second-order tensor, clearly Equation (7) will be valid for each of them. Hereafter we use R_{ij} to refer to any of the nine radiation tensors in general. Using Equation (7), the tranformation law for m_{ij}, J_{ij} and I_{ij} in matrix form can be written as

$$[m'] = [a][m][a]^T, \quad [J'] = [a][J][a]^T, \quad [I'] = [a]\,[I]\,[a]^T. \tag{9}$$

Equation (9) shows that, although m_{ij}, J_{ij} and I_{ij} are related to the same body and to each other through $m_{\alpha\beta}$ but in a rotated coordinate system, they can be obtained independently.

As a consequence of the transformation law (7), the component of any radiation tensor R_{ij} in the direction of an arbitrary unit vector $\mathbf{n} = n_i \mathbf{e}_i$ is $R_{ij} n_i n_j$. Also the first, second and third invariants of radiation tensor R_{ij} can be found from the following relations

$$I_1 = R_{ii}, \quad I_2 = \frac{1}{2}(R_{ij} R_{ij} - R_{ii} R_{jj}), \quad I_3 = \frac{1}{6}(R_{ii} R_{jj} R_{kk} - 3R_{ii} R_{jk} R_{jk} + 2R_{ij} R_{jk} R_{ki}). \quad (10)$$

In addition, since zeroth- and second-moment radiation tensors are symmetric tensors, their principal values are all real and the corresponding principal directions are mutually orthogonal to each other. Furthermore, the maximum (minimum) value of a tensor component is equal to the maximum (minimum) principal value. Moreover, if the immersed or floating body has a plane of symmetry, the direction perpendicular to that plane is a principal direction of radiation tensors and the other two principal directions lie in the plane of symmetry.

It should be mentioned that, in practice, it is more convenient to obtain radiation coefficients of a rotated body rather than those coefficients for a body in a rotated coordinate system. This can be done by considering the x_i and x'_i coordinate systems as inertia and body-fixed coordinate systems, respectively, such that before rotation x'_i coincides with the x_i coordinate system. If the radiation coefficients are known in the body-fixed coordinate system, we have for the rotated body

$$R_{ij} = a_{ki} a_{lj} R'_{kl}, \quad (11)$$

where a_{ki} can be assumed as the object rotation matrix.

3.2. Radiation tensors and improper orthogonal transformations

An orthogonal transformation, like the one governed by Equation (8), is defined to be a proper transformation when det (a_{ij}) is equal to $+1$ and an improper transformation when det (a_{ij}) is equal to -1. A proper transformation preserves the handedness of the coordinate system, whereas an improper transformation changes the handedness. A second-order tensor is a quantity that obeys the transformation law (7), whether the transformation is proper or improper. On the other hand, a psuedo-tensor is a quantity whose transformation law is similar to that of a tensor but has $\det(a_{ij})$ as a coefficient on its right-hand side. Therefore a pseudo-tensor differs from a tensor when the transformation is an improper one.

Now consider two rectangular Cartesian coordinate systems x_i and x'_i with the same origin where x'_i is obtained from x_i by an improper orthogonal transformation. Because the translational velocity is a first-order tensor (a polar vector) and the angular velocity is a first-order pseudo-tensor (an axial vector), under transformation of x_i to x'_i, we have

$$U_i = a_{ki} U'_k, \quad \Omega_i = -a_{ki} \Omega'_k. \quad (12)$$

A similar equation can be written by replacing free index i in Equation (12) with j and changing the dummy index k with l. Then it follows that $U_i U_j$ and $\Omega_i \Omega_j$ are components of two second-order tensors while $\Omega_i U_j$ and $U_i \Omega_j$ are components of two second-order pseudo-tensors. Now because T is a scalar and because the double-dot product of two second-order pseudo-tensors is a scalar, it can be deduced from Equation (3) that m_{ij} and I_{ij} are components of two tensors, while J_{ij} is the component of a pseudo-tensor, *i.e.*,

$$m'_{ij} = a_{ik} a_{jl} m_{kl}, \quad J'_{ij} = -a_{ik} a_{jl} J_{kl}, \quad I'_{ij} = a_{ik} a_{jl} I_{kl}. \quad (13)$$

To study the behaviour of added-mass and damping tensors of a floating body under improper orthogonal transformations, we refer to Equation (6). In this equation, F_i, U_i, and $\dot{U}_i$ are components of three polar vectors and M_i, Ω_i, and $\dot{\Omega}_i$ are components of three axial vectors. Therefore, $\widetilde{S}_{ij}$ and D_{ij} which map an axial\polar vector to a polar\axial vector are the components of two second-order pseudo-tensors; and $\widetilde{m}_{ij}$, $\widetilde{I}_{ij}$, B_{ij} and E_{ij} which map a polar\axial vector to another polar\axial vector are the components of four tensors. In other words, we can write

$$\widetilde{S}'_{ij} = -a_{ik}\,a_{jl}\,\widetilde{S}_{kl}, \quad D'_{ij} = -a_{ik}\,a_{jl}\,D_{kl}. \tag{14}$$

and

$$\begin{aligned} \widetilde{m}'_{ij} &= a_{ik}\,a_{jl}\,\widetilde{m}_{kl}, \quad \widetilde{I}'_{ij} = a_{ik}\,a_{jl}\,\widetilde{I}_{kl}, \\ B'_{ij} &= a_{ik}\,a_{jl}\,B_{kl}, \quad E'_{ij} = a_{ik}\,a_{jl}\,E_{kl}. \end{aligned} \tag{15}$$

Equations (13), (14) and (15) show that the zeroth-moment and second-moment radiation tensors obey the transformation law of a tensor while the first-moment radiation tensors obey the transformation law of a pseudo-tensor. If the components T_{ij} are used to refer to the components of one of the zeroth- or second-moment radiation tensors and the components P_{ij} are used to refer to the components of one of the first-moment radiation tensors, then Equations (13)–(15) can be summarized in matrix form as follows

$$[T'] = [a][T][a]^T, \quad [P'] = -[a][P][a]^T. \tag{16}$$

Equation (16) can be used to study the effect of body symmetries on the radiation tensors of an immersed or a floating body. Assume that a body has a plane of symmetry and the x_k-axis of the unprimed coordinate system is perpendicular to that plane. Now consider a primed coordinate system such that it is the reflection of unprimed coordinate system in the symmetry plane. Then the transformation symbol will be

$$a^k_{ij} = \begin{cases} -1 & : \quad i = j = k, \\ \delta_{ij} & : \quad \text{otherwise.} \end{cases} \quad : \quad k = 1, 2, 3, \tag{17}$$

where δ_{ij} is the Kronecker delta. It means that, apart from one of the leading diagonal elements which is -1, the rest of matrix $[a]$ is the same as the identity matrix. Therefore it follows that $\det(a_{ij}) = -1$, so the transformation defined by Equation (17) is an improper one. Hence, Equation (16) governs the transformation. On the other hand, due to the symmetry of the body, there must be no difference between the components of tensor **T** and pseudo-tensor **P** in primed- and unprimed-coordinate systems. In other words, due to the body symmetry, T_{ij} and P_{ij} remain invariant under the transformation (17). Consequently, Equation (16) takes the following form

$$[T] = [a][T][a]^T, \quad [P] = -[a][P][a]^T. \tag{18}$$

Now if Equation (17) is substituted in Equation (18), because only zero is equal to its additive inverse, it follows that some of the T_{ij} and P_{ij} components corresponding to a symmetric body must be zero. Observing the results of substitution of Equation (17) in Equation (18) for three cases of $k = 1, 2, 3$ reveals the following symmetry rules:

Let the x_k-axis, $(k = 1, 2, 3)$, be perpendicular to the symmetry plane of the body then:

1. For matrix $[T]$, corresponding to a tensor **T**, all off-diagonal elements in the kth-row and kth-column are zero.

2. For matrix $[P]$, corresponding to a pseudo-tensor $\mathbf{P}$, only off-diagonal elements in the kth-row and kth-column are non-zero.

These rules can be used to find the zero components of tensors and pseudo-tensors corresponding to a body with 1, 2 or 3 orthogonal symmetry planes without doing the matrix multiplications of Equation (18). For other kinds of symmetry a_{ij} will be defined by an equation different from Equation (17) but Equation (18) is still valid. The final results are given in the literature [see for instance, 1, pp. 183–192].

3.3. Parallel-axes theorem for radiation tensors

Consider two coordinate systems x_i and x_i' both fixed with respect to a body at O and O' such that the corresponding axes of the two coordinate systems are parallel. If U_i and U_i' denote the components of the translational velocity of the body at O and O', respectively, and Ω_i is the component of the angular velocity of the body, then because O and O' can be assumed as two points of the rigid body and because $\mathbf{e}_i' = \mathbf{e}_i$, one can write

$$U_i = U_i' - \epsilon_{ijk}\Omega_j d_k \,, \tag{19}$$

where ϵ_{ijk} is the component of the alternator tensor and d_k is the component of the position vector of O' with respect to O. Introducing the anti-symmetric tensor components $H_{ij} = -\epsilon_{ijk} d_k$, we observe that Equation (19) takes the following forms

$$U_i = U_i' + H_{ik}\Omega_k \quad \text{or} \quad U_j = U_j' + H_{jl}\Omega_l \,. \tag{20}$$

Substituting for U_i and U_j from Equation (20) in Equation (3) and taking into account that T is an invariant, and that U_i' and Ω_i are arbitrary and generally non-zero, we obtain the translation law for the tensor components m_{ij}, J_{ij} and I_{ij} which in matrix form can be written as follows:

$$\left[m'\right] = [m], \quad \left[J'\right] = [m][H] + [J], \quad \left[I'\right] = [H][m][H]^T + [J]^T[H] + [H]^T[J] + [I]. \tag{21}$$

These equations can also be expressed in terms of tensor components in the primed coordinate system, *i.e.*,

$$[J] = [m'][H]^T + [J'], \quad [I] = [H][m'][H]^T + [J']^T[H]^T + [H][J'] + [I']. \tag{22}$$

The last equation is similar to that of Happel and Brenner [1, Formula 5–4.10]. For a floating body, if one derives from Equation (6) the scalar quantities F_iU_i and $M_i\Omega_i$ and substitutes for U_i and U_j from Equation (20), then, since the total power $F_iU_i + M_i\Omega_i$ is an invariant, a transformation law similar to Equation (21) will be obtained for added-mass and damping tensors. In summary, if $\mathbf{R}^0$, $\mathbf{R}^1$ and $\mathbf{R}^2$ are used to denote zeroth-, first- and second-moment radiation tensors in general, then the translation law for radiation tensors of a body, immersed or floating, can be written as

$$\mathbf{R}'^0 = \mathbf{R}^0, \quad \mathbf{R}'^1 = \mathbf{R}^0\cdot\mathbf{H} + \mathbf{R}^1, \quad \mathbf{R}'^2 = \mathbf{H}\cdot\mathbf{R}^0\cdot\mathbf{H}^T + \mathbf{R}^{1^T}\cdot\mathbf{H} + \mathbf{H}^T\cdot\mathbf{R}^1 + \mathbf{R}^2. \tag{23}$$

Equation (23) shows that only zeroth-moment radiation tensors, which are independent of the choice of coordinate system are truly second-order tensors. The first-moment and second-moment radiation tensors are not precisely tensor quantities, since they are dependent on the position of the origin of coordinate system. One may call $\mathbf{R}^1$ and $\mathbf{R}^2$ generalized tensors.

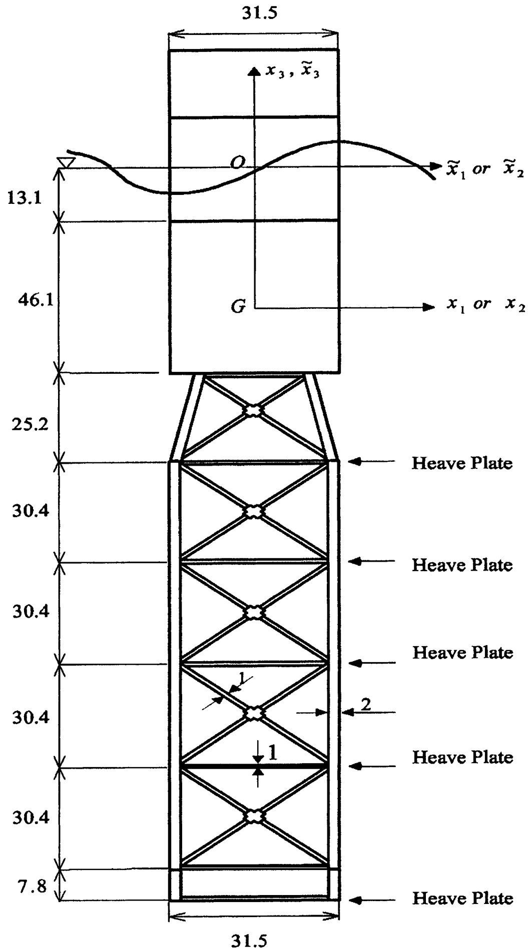

Figure 1. Sketch of truss-spar platform.

4. Application of transformation method

In order to show the application of the transformation method in the response analysis of offshore structures we shall consider a truss-spar platform as shown in Figure 1. A 1:100 scaled model of this platform has been the subject of few experimental studies [8,9]. In Figure 1 the x_1x_3- and x_2x_3-plane of coordinate system are planes of symmetry. Therefore, using symmetry rules of Section 3.2, it can be shown that, in the context of a linear radiation-diffraction model, both heave and yaw are independent of all the other five degrees of freedom; surge and pitch are dependent only on each other; and so are sway and roll. We shall consider the motion of the platform in heave, surge and pitch.

In the context of a linear theory, it is assumed that the platform is composed of two separate bodies, i.e., a surface-piercing cylindrical hull and an immersed truss. The hydrodynamic interactions between the hull and the truss, and among the various elements of the truss, are assumed to be of second- and higher-order, and therefore neglected. Moreover, it is well-known that in deep water the wave motion decays rapidly in depth, therefore as an approximation in the radiation problem, because the truss is far below the free surface, the truss is modelled as a body oscillating in an unbounded fluid. Consequently, the added-mass coefficients of the truss are assumed to be independent of frequency. In addition, it is assumed

Table 1. Natural periods and standard deviations of the truss spar.

	Natural periods (sec)		Standard deviations[a] (m or deg)		
	Estimated	Measured[b]	Estimated	Measured[b]	Simulated[b]
Heave	24·4	25·0	0·545	0·531	0·3659
Pitch	62·4	64·4	0·913	1·287	0·8821
Surge	511	510	1·959	—	—

[a] For JONSWAP wave spectrum with $H_{1/3} = 15\,\mathrm{m}$ and $T_0 = 15\,\mathrm{sec}$.
[b] Stansberg *et al.* [9].

that the diameter of the hull is not too large to radiate significant waves. Therefore, taking also into account that the natural frequencies of a spar platform are relatively low, the radiation damping due to the hull motions is neglected and the added-mass coefficients of the hull are assumed to be independent of frequency.

As shown in Figure 1, the truss section has four identical bays. Excluding the heave plates, which are placed at the bottom of each bay, all substructures of the truss are made of circular cylindrical members. Therefore, their added-mass coefficients can be obtained by transformation of the coefficients of a typical horizontal circular cylinder. Using this technique, the added-mass coefficients of one bay of the truss about its local axes is obtained. The results will be valid for the other three identical bays. Then, the parallel-axes theorem is used to derive the coefficients of each bay about the origin of the global coordinate system of the platform at its centre of gravity. The same method is used to add the contributions of the remaining parts of the truss, the heave plates and the hull to obtain the added-mass coefficients of the whole structure. The added-mass coefficients obtained from this method can be used to calculate the natural periods of the platform. In Table 1 the results of this calculation are compared by the measured values reported by Stansberg *et al.* [9]. The good agreement obtained between the calculated and measured results indicates that the transformation approach is valid and can achieve good accuracy.

As the above example indicates, in contrast to the common method of Morison's equation [10] and normal-component approach, it is not necessary to apply the transformation method to each and every element of the structure. In other words, when substructure B can be produced by rotating, reflecting or translating substructure A, the radiation coefficients of B can be obtained from those of A by using the appropriate transformation law, rather than from direct calculations. This reduces the amount of calculations greatly.

This advantage becomes more significant if the wave excitation and viscous forces acting on the truss spar can also be obtained by a method different from Morison's equation. With this purpose and based on the transformation method, a model was proposed and used [11] for the dynamic-response analysis of the truss spar in Figure 1. Details of the calculations and the model are given in [11]. The estimated motions obtained from this model are given in Table 1 where the measured and simulated results of Stansberg *et al.* [9] are also included. It can be seen that the transformation method can efficiently be applied for dynamic-response analysis of truss-spar platforms.

5. Conclusions

Each of the 6×6 radiation matrices associated with an immersed or a floating body can be partitioned into four 3×3 sub-matrices. The diagonal sub-matrices correspond to symmetric second-order tensors. The off-diagonal sub-matrices are the transposes of each other and correspond to a second-order pseudo-tensor. As a result, transformation laws of a second-order tensor in rotation, reflection and translation of coordinate systems can be applied for radiation coefficients. The dynamic-response analysis of offshore structures like truss-spar platforms can be performed by the transformation method. One may investigate if this method can deliver the same level of accuracy and efficiency for other types of offshore structures.

References

1. J. Happel and H. Brenner, *Low Reynolds Number Hydrodynamics with Special Applications to Particulate Media*. New Jersey: Printice-Hall Inc (1965) 553 pp.
2. A.I. Borisenko and I.E. Taparov, *Vector and Tensor Analysis with Applications*. New Jersey: Printice-Hall Inc (1968) 257 pp.
3. J.N. Reddy and M.L. Rasmussen, *Advanced Engineering Analysis*. New York: John Wiley and Sons (1982) 488 pp.
4. L.E. Malvern, *Introduction to the Mechanics of a Continuous Medium*. New Jersey: Printice-Hall Inc (1969) 713 pp.
5. G.B. Arfken and H.J. Weber, *Mathematical Methods for Physicists*. San Diego: Academic Press (2001) 1112 pp.
6. L. Landweber and C.-S. Yih, Forces, moments and added masses for Rankine bodies. *J. Fluid Mech.* 1 (1956) 319–336.
7. J.N. Newman, *Marine Hydrodynamics*. Cambridge, Massachusetts: The MIT Press (1977) 402 pp.
8. M.J. Downie, J.M.R. Graham, C. Hall, A. Incecik and I. Nygaard, An experimental investigation of motion control devices for truss spars. *Marine Struct.* 13 (2000) 75–90.
9. C.T. Stansberg, I. Nygaard, H. Ormberg, M.J. Downie, A. Incecik and J.M.R. Graham, Deep-water truss spar in waves and current—experiments vs. time–domain coupled analysis. *Deep Offshore Technology Conference*. Rio de Janeiro (2001).
10. J.R. Morison, M.P. O'Brien, J.W. Johnson and S.A. Schaaf, The forces exerted by surface waves on piles. *Petrol. Trans. AIME* 189 (1950) 140–157.
11. K. Sadeghi, A. Incecik and M.J. Downie, Response analysis of a truss spar in frequency domain. *J. Mar. Sci. Technol.* 8 (2004) 126–137.